Hilroy

80 Pages

27.6 cm x 21.2 cm

Ruled 7 mm • Ligné 7 mm

EXERCISE BOOK
CAHIER D'EXERCICES

NAME/NOM ~~STAT 321~~

SUBJECT/SUJET STAT 321

ASSEMBLED IN CANADA WITH IMPORTED MATERIALS
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

Lecture 1

1/8/20

$\boxed{SS10}$

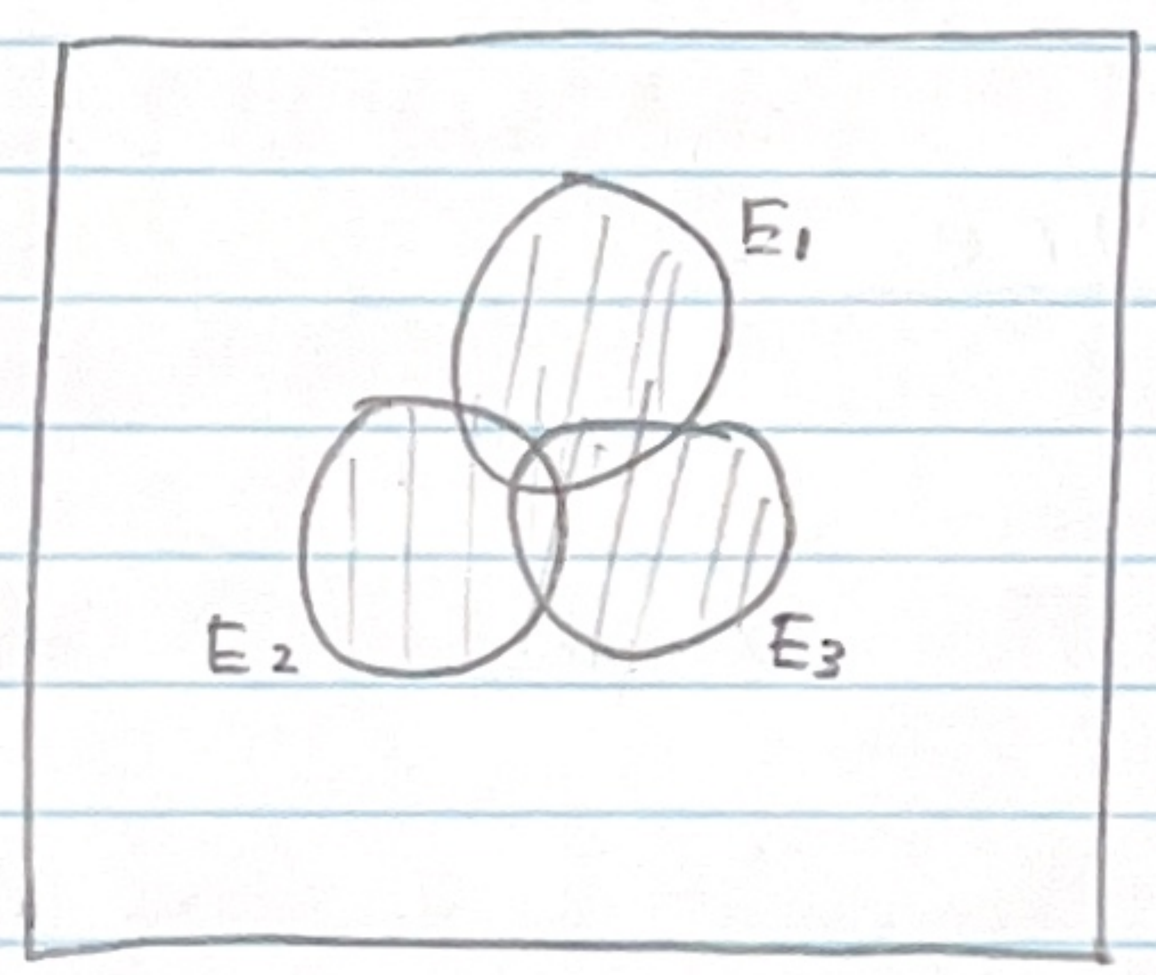
*need to subtract overlapping portions to only account for it once

$\boxed{SS\ 12\ Q2}$

By combinatorial formula:

$$C_r = \frac{15!}{5!(15-5)!} = 3003$$

$\boxed{SS\ 13\ Q3}$



*Must apply inclusion exclusion formula

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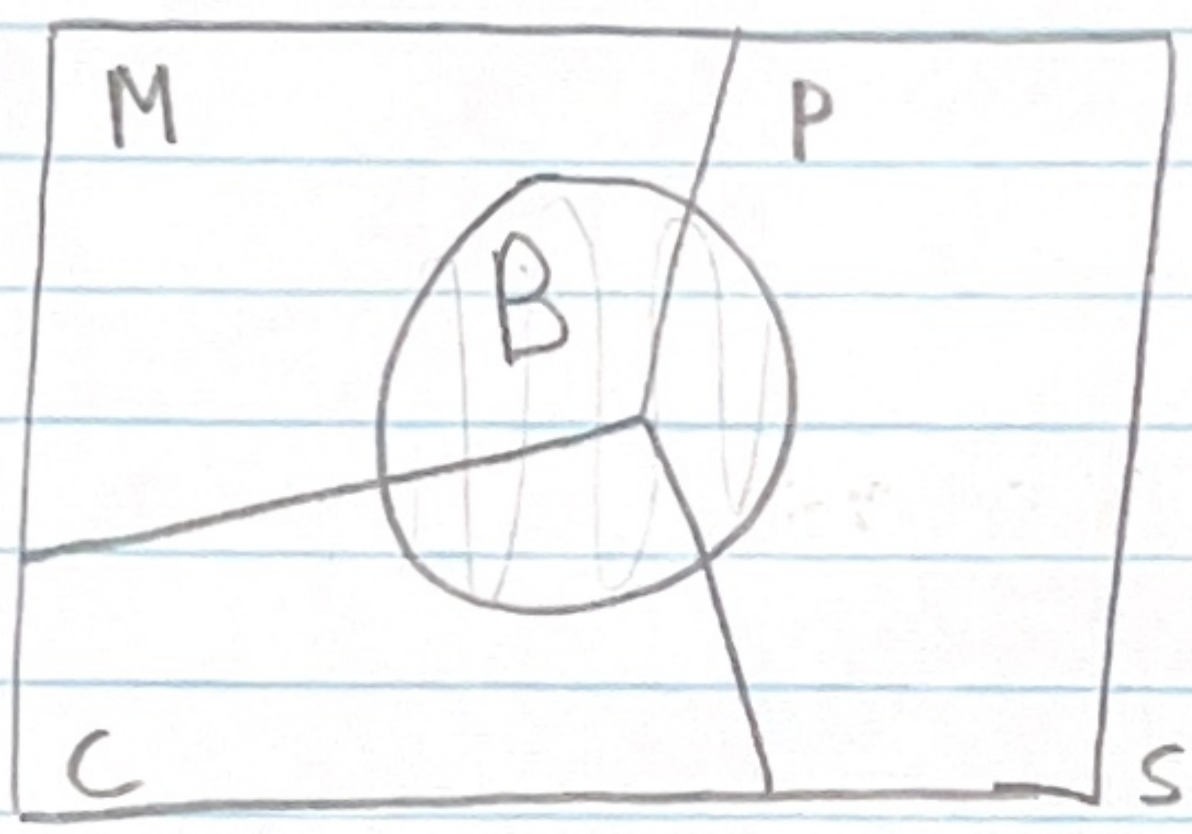
Lecture 2

Conditional probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$\boxed{\text{SS11 Q1}}$



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Lecture 3

Review:
- Module 1 + Exercises
- Module 2 + Exercises
- Module 3 → SS " "
- HW 1, 2

$\boxed{\text{SS 17}}$

"Toss a biased coin with 30% chance of heads, and you toss it 10 times"

Define success event = toss a heads

Tutorial 1

$$P(A \cap B) = \underbrace{P(A) \cdot P(B)}_{\text{Independent}}$$

$$P(A \cap B) = \underbrace{P(A|B) \cdot P(B)}_{\text{Dependent}}$$

$\boxed{\text{Ex}}$ "A box contains three 'good' cards and two 'bad' cards. Player A chooses one card and then player B chooses a card"

① Find $P(A \text{ good})$

SOLUTION → $P(A \text{ good}) = 3/5$

② Find $P(B \text{ good} \mid A \text{ good}) = 2/4$

③ Find $P(B \text{ good} \mid A \text{ bad}) = 3/4$

④ Find $P(A \cap B) = P(B|A)\, P(A) = (2/4)(3/5) = 3/10$

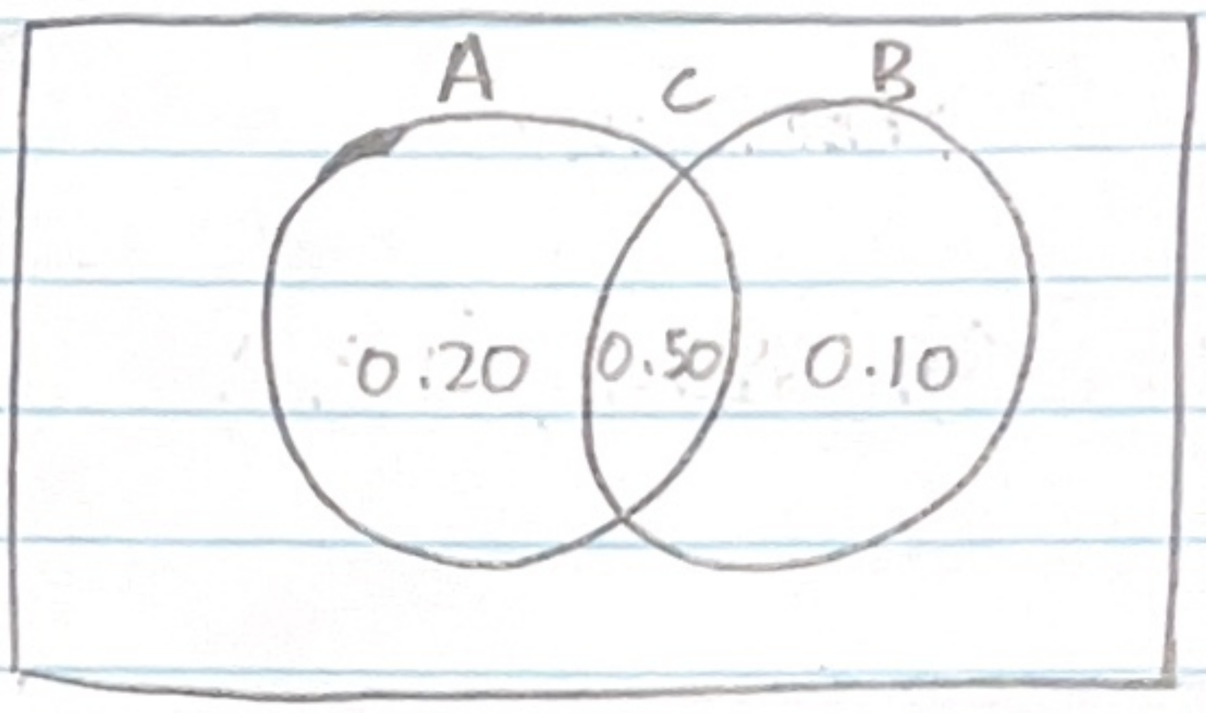
⑤ Find $P(B) = P(B \cap A) + P(B \cap A')$ [LOTP]

$\boxed{\text{Ex}}$ "n people in a room, odds they share the same birthday"

$$P(\text{nobody sharing birthday}) = \frac{\overbrace{365 \cdot 364 \ldots}^{n \text{ times}}}{365^n} = \frac{365!}{(365-n)! \cdot 365^n}$$

Ex "A track start runs two races, the probability is:
[A] Wins the first race = 0.70
[B] wins the second race = 0.60
[C] wins both races = 0.50

(a) Find probability he wins one race



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.70 + 0.60 - 0.50$
$= \boxed{0.80}$

(b) wins exactly one race

$P = 0.20 + 0.10 = \boxed{0.30}$

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Lecture 4

SS 25 QC
1 − P(none) easier than P(1 defective) + P(2 defective).....

Poisson Process (If applicable)
① Rule I
② Rule II
③ Rule III

$P(E) = \underbrace{\lambda}_{\text{rate}} \overbrace{L}^{\text{interval length}}$ (For small L)

SS31 Q4

$\lambda = 4$ (per minute)
$L = 0.5$ (per minute)

Lecture 5

(Module 3 Part II)

Cumulative distribution function: $F(X)$

$$F(X) = \int_{-\infty}^{x} f(t)\,dt$$

Probability density function: $f(x)$

$$f(x) = F'(x) = \frac{d}{dx} F(X)$$

Quiz #1 → Polynomial/exponential functions

Expected Value of X: (mean) (for continuous)

$$E(X) = \int_{-\infty}^{\infty} x \underbrace{f(x)}_{\text{pdf}} dx$$

Formulas → Binomial/Poisson distributions
[Quiz #1]

Quiz #1 → up to (Module 3 Part II) SS13

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Lecture 6

How to find pdf from cdf?

↳ Take the derivative
$f(x) = F'(x)$

Cumulative distribution Function:

$P(X \leq 50) = F_X(50)$

Normal Distribution:

Standardizing a random variable (X)

↳ $Z = \frac{X - \mu}{\sigma}$

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Lecture 7

Exponential Distribution: $X \sim Exp(\lambda)$ [X follows exponential distribution]

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

cdf → $F(x) = 1 - e^{-\lambda x}$ for $x \geq 0$

$E(X) = \mu = \frac{1}{\lambda}$

$Var(X) = \frac{1}{\lambda^2}$

$M_X(t) = \frac{\lambda}{\lambda - t}$

Exponential random variable is memoryless:

$P(X > 10+2 \mid X > 10)$

$= P(X > 2)$

"Probability it will last 2 more hours is independent of how many hours it has already run"

memoryless $\rightarrow$ Failure Rate is Constant

Failure Rate: (General)

$$\lambda(x) = \frac{f(x)}{1-F(x)} = \frac{pdf(x)}{1-cdf(x)}$$

20 Lecture 8

(Module 4a)

$\boxed{\text{SS 34}}$ $\rightarrow$ START

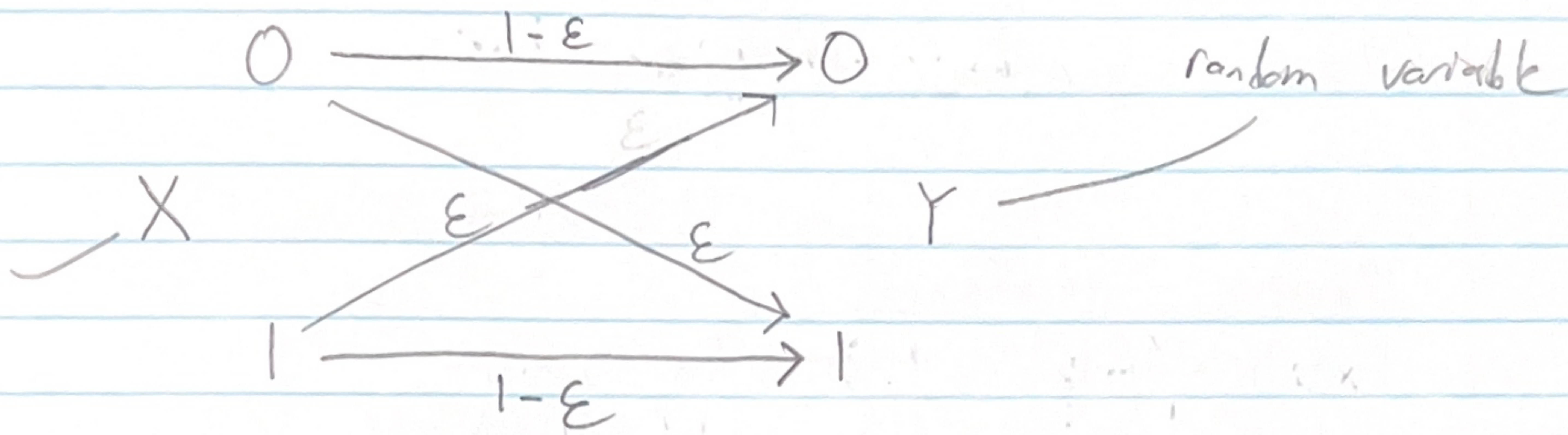
$\boxed{\text{SS 36}}$ $\rightarrow$ Last page for Quiz #2

Conditional Distribution:

$P(\overbrace{X=x}^{A} \mid \overbrace{Y=y}^{B}) \rightarrow \boxed{\text{Remember}}$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Part II

<u>Data communication over unreliable channels:</u>



$\varepsilon \to$ probability of error

$\boxed{\text{parity bits}} \to$ redundant bits (identical) to improve reliability

Machine Learning: a probabilistic perspective $\to$ Kevin Murphy

Lecture 1

Ex: $\int_{\mathbb{R}^n} g(x)\,dx \to$ Monte-Carlo method

Lecture 2

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2/28/20

Generating Random Variables:

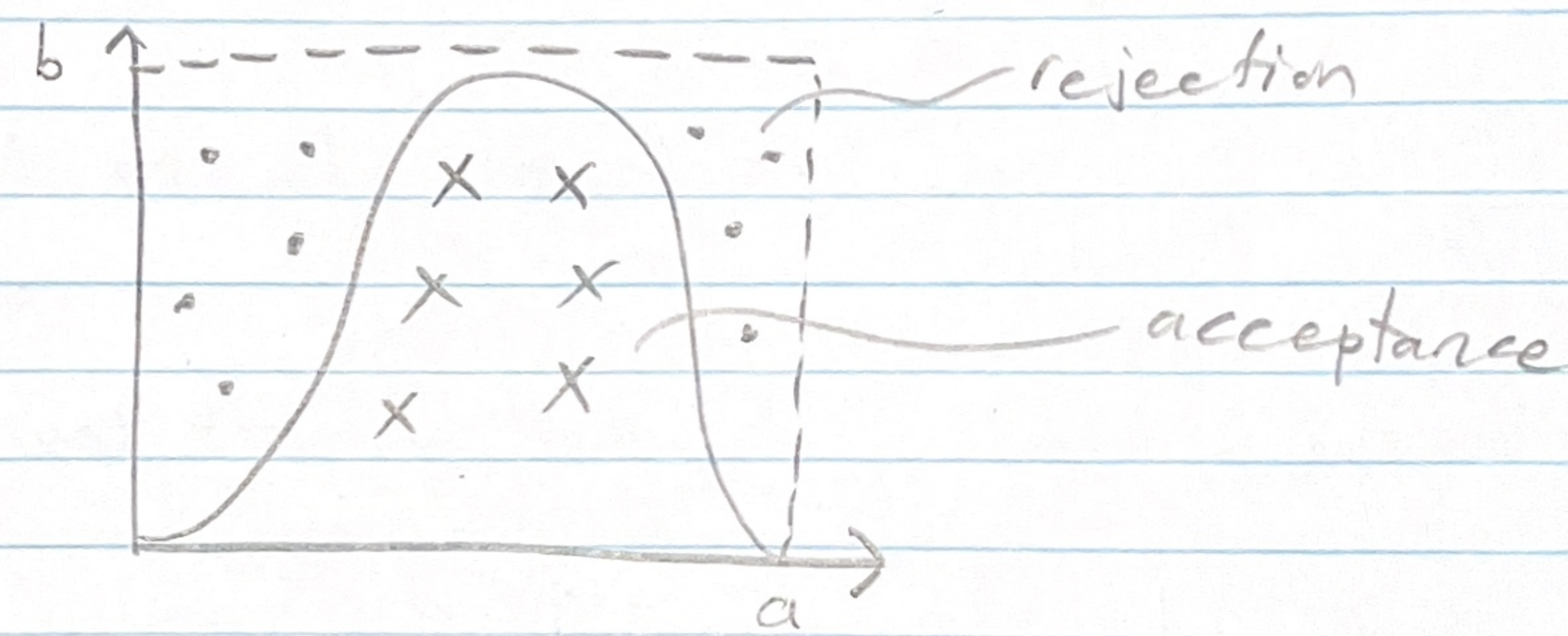
① Inverse Transform Algorithm
② Polar algorithm for guassian RV

Inverse Transform:

cdf

$F_X(x) = 1 - e^{-\lambda x}$, $\quad y = F_X(x) = 1 - e^{-\lambda x}$

$x = \frac{-1}{\lambda} \log(1-y) = F_X^{-1}(y)$



Tutorial 1

Lecture 3

How to measure uncertainty:

$I(X=k) = \log_2(1/p_k) = -\log_2(p_k)$ bits

Axioms of uncertainty:

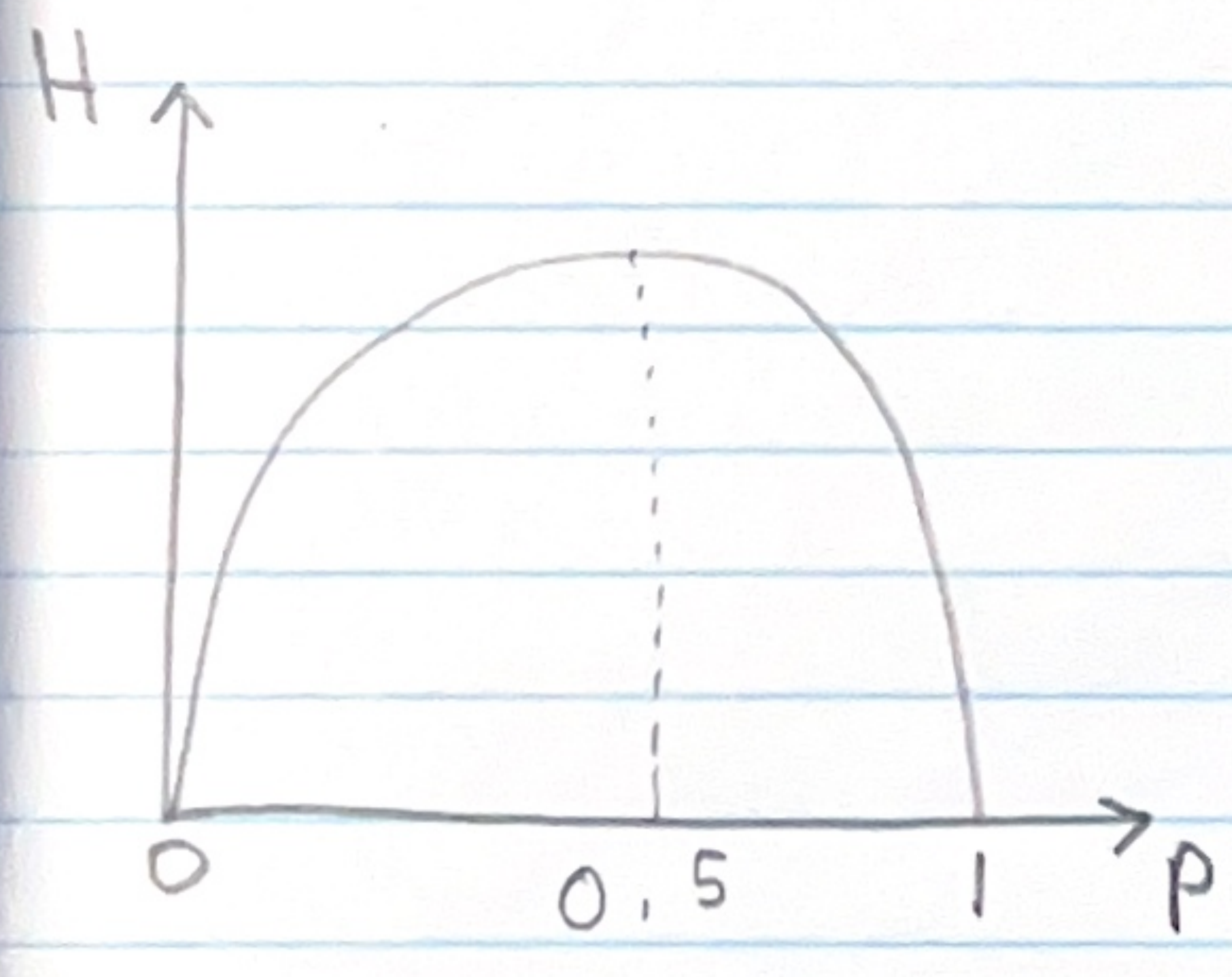
1. $I(X=k) > I(X=m)$ if $p_k < p_m$
2. $I(X=k) \to 0$ as $p_k \to 1$
3. $I(X=k) \geq 0$
4. X_1, X_2, are independent then: $I(X_1=k, X_2=m)$
   $= I(X_1=k) + I(X_2=m)$

If all cases are equally likely:

$H_X = \log_2(K)$ [Upper Bound]

Tutorial 2

$\log_2(\frac{1}{p}) =$ Bits of information



$\boxed{\text{Ex:}}$ $X \sim \{1, 2, 3, 4, 5, 6\}$
$Y \sim \{1, 2, 3, 4, 5, 6\}$ $Y \leq X$

$$H(y) = E\left[\log\left(\frac{1}{p(y)}\right)\right] = \sum p(y) \log\left(\frac{1}{p(y)}\right)$$

$X = 1 \rightarrow y = 1$
$X = 2 \rightarrow y = 1, 2$
$X = 3 \rightarrow Y = 1, 2, 3$